0

$$\|L_1(t_1) - L_2(t_2)\|^2$$

$$= \|X\|^2 t_1^2 + \|x\|^2 t_2$$

$$- 2 t_1 t_2 \, X \cdot x + 2 t_1 \, X \cdot (Y - y) - 2 t_2 \, x \cdot (Y - y)$$

$$+ \|Y - y\|^2$$

$\cancel{P = A t_1 +}$

$$x = \begin{matrix} t_1 \\ t_2 \end{matrix}.$$

$$P_1(t_1) = X t_1 + Y \qquad Ax = b.$$

$$P_2(t_2) = x t_2 + y$$

$$\|P_1 - P_2\| = \|X t_1 + Y - x t_2 - y\|^2$$

$$= \left\| \begin{pmatrix} X & \\ & -x \end{pmatrix} \begin{pmatrix} t_1 \\ t_2 \end{pmatrix} \right.$$

$$\begin{pmatrix} X & t_1 \\ -x & t_2 \end{pmatrix}$$

$$f(t_1,t_2) = At_1^2 + Bt_2^2 + Ct_1t_2 + Dt_1 + Et_2 + F$$

$$\begin{cases} \dfrac{\partial f}{\partial t_1} = 2At_1 + Ct_2 + D \\[2ex] \dfrac{\partial f}{\partial t_2} = 2Bt_2 + Ct_1 + E \end{cases}$$

$$\left.\begin{aligned} \frac{\partial^2 f}{\partial t_1^2} &= 2A \\ \frac{\partial^2 f}{\partial t_2^2} &= 2B. \\ \frac{\partial^2 f}{\partial t_1 \partial t_2} &= C \end{aligned}\right\} \qquad H = \begin{pmatrix} 2A & C \\ C & 2B \end{pmatrix}$$

H PSD ??

$$\Rightarrow \quad P(\lambda) = \operatorname{Det}(H - \lambda I)$$

$$= \begin{vmatrix} 2A-\lambda & C \\ C & 2B-\lambda \end{vmatrix}$$

$$= (2A-\lambda)(2B-\lambda) - C^2$$

$$P(\lambda) = \lambda^2 + \lambda(-2B - 2A) + 4AB - C^2$$

$$= \lambda^2 - 2\lambda(A+B) + 4AB - C^2$$

$A^2 - B^2 = (A-B)(A+B)$

$\cancel{B^2 - C^2 = (A+B)(A-1}$

$A^2 \cancel{4AB} - B^2$

$$\Delta = [-2(A+B)]^2 - 4(4AB - C^2)$$

$$= 4(A+B)^2 - 16AB + 4C^2$$

$$= 4(A^2 + 2BA + B^2) - 16AB + 4C^2$$

$$= 4(A^2 - 2BA + B^2) + 4C^2$$

$\boxed{\Delta = 4(A-B)^2 + 4C^2} > 0$ Always 2 real solution [Hermitian!!]

$\cancel{= 4[(A-B+C)(A-B+C)]}$

$\cancel{\Delta = 4(A-B-C)(A-B+C)}$ $\cancel{\Delta \geq 0 ??}$

$\cancel{\Delta \geq 0}$

$A = X^2$

$B = x^2$

$C = -2X \cdot x$

$\cancel{\Delta = 4(X^2 - x^2 + 2X x)(X^2 - x^2}$

$$\lambda_1 = \frac{1}{2}\left\{ +2(A+B) - \sqrt{\Delta} \right\} \geqslant 0 \ ?$$

$$\lambda_2 = \frac{1}{2}\left\{ 2(A+B) + \sqrt{\Delta} \right\} \geqslant 0 \ ?$$

$$\cancel{2\lambda_1 = 2(A+B) + 2\sqrt{(A-B)^2 + C^2}}$$

$$\cancel{= 2(X^2+x^2) + 2\sqrt{(X^2-x^2)^2 + 2X.x}}$$

$$X.x = X_1 x_1 + X_2 x_2 + X_3 x_3.$$

$$\frac{1}{4}\Delta = (X^2 - x^2)^2 + 4(X.x)^2 \qquad (X.x)^2 : X$$

$$2\lambda_2 = 2(X^2+x^2) + \sqrt{\Delta} \geqslant 0. \quad !$$

$$2\lambda_1 = 2(X^2+x^2) - \sqrt{4(X^2-x^2)^2 + 16(X\cdot x)^2} \overset{?}{\geqslant} 0$$

$$2(X^2+x^2). \overset{?}{\geqslant} \sqrt{4(X^2-x^2)^2 + 16(X.x)^2}$$

$$4(X^2+x^2)^2 \overset{?}{\geqslant} 4(X^2-x^2)^2 + 16(X.x)^2$$

$$4\left\{ (X^2+x^2 - X^2+x^2)(X^2+x^2+X^2-x^2) \right\} \overset{?}{\geqslant} 16(X.x)^2$$

$$(2x^2)(2X^2) \overset{?}{\geqslant} 4\,(X.x)^2$$

$$A \quad X^2 x^2 \overset{?}{\geqslant} (X.x)^2 \longrightarrow B$$

$$\|X\|^2 : \quad X_1^2 + X_2^2$$

$$\|x\|^2 = \quad x_1^2 + x_2^2 .$$

$$A \Rightarrow (X_1^2 + X_2^2)(x_1^2 + x_2^2) \overset{?}{\geqslant} (X_1 x_1 + X_2 x_2)^2$$

$$\cancel{X_1^2 x_1^2} + \cancel{X_2^2 x_2^2} + X_1^2 x_2^2 + X_2^2 x_1^2 \overset{?}{\geqslant}$$

$$\cancel{X_1^2 x_1^2} + \cancel{X_2^2 x_2^2} + 2X_1 x_1 X_2 x_2 .$$

$$X_1^2 x_2^2 + X_2^2 x_1^2 - 2\,X_1 x_1 X_2 x_2 \overset{?}{\geqslant} 0$$

$$\boxed{(X_1 x_2 + X_2 x_1)^2 \overset{?}{\geqslant} 0 \qquad \text{YES!}}$$